ON SPECTRAL THEORY OF LINEAR OPERATORS

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INTRODUCTION

Spectral Theory is in its best form when one considers normal operators in Hilbert spaces. However, for finite dimensional spaces one has the reduction to Jordan's Canonical form. In an attempt to generalise this reduction for arbitrary Banach spaces, Dunford introduced the concept of spectral operators (see [4,5 and 6] for the details). The pioneering work of Dunford has inspired several generalisations of this concept, for example, see [2,3,11,12,17 and 22].

While formulating the concept of spectral measure, Dunford observes in passing that the Spectral Reduction theory can be given a fair degree of precision even without assuming that the spectral projectors are bounded. None the less until very recently all the works concerned with Spectral Reduction problems (i.e. leaving out those which take an operational calculus as the starting point e.g. [3,11 and 12]) consider the case where spectral projectors are all bounded. The only exception to this seems to be the recent work of Bartle [1], where the spectral projectors turn out to be closed.

In the second part of this dissertation we consider a certain class of operators induced by Branching Processes, and study them in full details. Here we encounter a situation where the spectral projectors are not necessarily bounded. So in Part I we develop an abstract theory to cope with this situation.

It turns out, as a consequence of the restrictions we impose, that the spectral projectors we consider are all closable. We have not yet studied the connection between our work and that of Bartle.

The Bibliography is by no means complete. We refer the reader to volume III of the encyclopaedic book of Dunford and Schwartz for a complete list of references in our field of study and allied fields. For the most part we have followed the notations and conventions of [7].