Jurnal Teknologi

Full paper

# Variance Corrected Proportional Hazard Model for the Analysis of Recurrent Multiple Failure Modes

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**Article history**

Received :21 January 2013
Received in revised form:
7 May 2013
Accepted :25 June 2013

**Graphical abstract**

$h_{ik}(t) = h_o(t)\exp(\beta' Z_{ik})$

**Abstract**

Automobiles do fail repetitively owing to different types of failures. Modeling of such products should concern dissimilar recurrent failure types. Failure types are habitually correlated to each other. This study has utilized *variance corrected proportional hazard models (VCPH)* for modeling multiple failure occurrences of different failure types of automobiles taking into account the correlated nature of the failures. Though original Cox proportional hazard (PH) models require failure events to be independent, VCPH can handle non independency. In this study, this model is utilized for the analysis of multi-type, multiple occurrences of failures in automobiles, where the assumption of independence among failure times is violated. The VCPH model obtains parameter estimates by first fitting a Cox PH model that ignores the dependence structure and then replaces the naive standard errors with estimates from empirical sandwich variance estimation in order to incorporate the non-independence of times between failures. This study applies the Prentice, Williams and Petersen (PWP) models to model multiple occurrences of different failures in automobiles as proportionality among failure events were well demonstrated when risk interval is taken as 'gap time' and since PWP models are specified with a gap time risk interval. The Information Matrix (IM) test of White is applied for the checking of the PH model specification with multivariate failure time data. White's paper on inference from missspecified models presented the IM test as a test for correct model specification. The objective of the study was to find out how automobile type and type of failure affect the time to failure. Applying the best suitable VCPH model to the data, it was revealed that both automobile and failure type have an impact on timing of failure however this effect doesn't change over multiple failure occurrences of the automobile.

*Keywords*: VCPH models; multiple type failures; automobile failures; PWP model; information matrix test

**Abstrak**

Kereta gagal secara berulang bergantung kepada jenis kegagalan. Pemodelan produk tersebut harus diberi perhatian yang berbeza terhadap jenis kegagalan yang berulang. Jenis kegagalan adalah lazimnya berkait rapat antara satu sama lain. Kajian ini telah menggunakan model berkadaran bahaya yang diperbetulkan (VCPH) untuk memodelkan kejadian kegagalan pelbagai dengan mengambil kira sifat hubungan kegagalan tersebut. Walaupun model asal cox bahaya berkadaran (PH) memerlukan peristiwa kegagalan untuk menjadi bebas, VCPH boleh mengendalikan bukan ketidakbersandaran. Dalam kajian ini, model ini digunakan untuk analisis pelbagai, kepelbagaian kejadian kegagalan dalam kereta, di mana andaian ketidakbersandaran antara masa kegagalan dilanggar. Model VCPH mendapat anggaran parameter dengan pemasangan pertama model COX PH yang mengabaikan struktur bersandaran dan kemudian menggantikan ralat piawai yang naïf dengan anggaran daripada anggaran varian emperik sandwich untuk menggabungkan bukan ketidaksandaran masa antara kegagalan. Kajian ini menggunakan Prentice, William and Petersen (PWP) model untuk memodelkan kejadian pelbagai daripada kegagalan yang berbeza di dalam kereta sebagai perkadaran antara peristiwa kegagalan adalah menunjukkan apabila selang risiko diambil sebagai 'jurang masa' dan model PWP dispesifikasikan dengan jurang masa selang risiko. Maklumat Matrik (IM) ujian White digunakan untuk memeriksa spesifikasi model PH dengan data masa multivariate kegagalan. Kesimpulan kertas White daripada model tidak tepat dibentangkan ujian IM sebagai ujian bagi spesifikasi model yang betul. Objektif kajian ini adalah untuk mengetahui bagaimana jenis kereta dan jenis kegagalan yang mempengaruhi masa kegagalan. Penggunaan model VCPH yang paling sesuai untuk data, menunjukkan bahawa kedua-dua faktor jenis kereta dan jenis kegagalan mempunyai kesan ke atas masa kegagalan. Walau bagaimanapun kesan ini tidak berubah terhadap pelbagai kegagalan kereta.

*Kata kunci*: Model VCPH; kegagalan pelbagai cara; kegagalan kereta; model PWP; ujian maklumat matriks

## 1.0 INTRODUCTION

There is a large scale impact of warranty policies on product sales, brand reputation and competitiveness in today's business world. Especially in the automobile industry, manufacturers are currently facing huge product recalls, increased warranty claim rates that lead to sinking profits, and loss of market capitalization. Failures over the warranty period are closely linked to product reliability. Automobiles fail repetitively in different failure modes and this study focus on modeling those recurrent multiple failures modes. The primary objective of this study is to explore an empirical application of analyzing recurrent multiple mode failure time data without incorporating the assumption of independence among failure events. The application of the study includes recurrent failure occurrences in automobiles manufactured by a particular company and the effect of the type of automobile and failure mode occurred on the failure event time is evaluated. Thereby a clear understanding of the type of automobile and failure mode on failure event timing can be achieved.

The methods of analyzing failure type data can be basically classified into parametric and non-parametric methods. The correlated structure of multiple failure time data makes it complicated to use parametric models to model recurrent multiple mode failure time data. Therefore, non-parametric methods which require no distributional assumptions would be more suitable in this context. The variance-corrected proportional hazards (VCPH) model proposed by the authors is an extension of the Proportional Hazard (PH) model, which takes into account the lack of independence between failure times. The VCPH is semi-parametric in the sense that no assumption is made about the distribution of failure times and the only assumption made is that of proportionality of hazards between the levels of the covariates.

These models obtain parameter estimates by first fitting a Cox PH model that ignores the dependence structure and then replace the naive standard errors with estimates from an empirical sandwich variance estimator in order to incorporate the non-independence of times between failures. This study applies the VCPH models to model recurrent multiple mode failure of automobiles. In this study the time (mileage) to failure of an automobile product is modeled considering selection of failure types that can occur for three different types of automobiles manufactured by the company and six types of failures are considered. The PH model can include the effect of covariates in the reliability function. This model requires two assumptions, namely, the proportionality of hazard rates between the different levels of the covariates and the independence of failure times. Peña *et al.*[11] proposed a general class of models for repairable systems, which comprise a general synthesis of several repairable systems models such as the modulated renewal process of Cox[3], extended Cox PH model considered by Prentice *et al.*[12] and the well-known VCPH models of: (i) Anderson–Gill (AG); (ii) Wei, Lin and Weissfeld (WLW); and (iii) Prentice, Williams and Peterson (PWP) as reported in Therneau *et al*.[14]. Ezzell *et al.* [6] have discussed about variance-corrected proportional hazards models that have been developed by statisticians taking into account a lack of independence among failure times. There is considerable controversy about how to model multiple failure time data, especially ordered data, and thus they have offered general guidelines for choosing among these models. Analyzing several failure modes separately and merging has been suggested by Doganaksoy *et al.* [5]. Analyzing by individual failure modes is not possible for every situation often because of lack of data or because the failure modes are not independent. Therefore in this study correlation between, failure types and failure events will be considered.

To assess the goodness of the model fitted, Information Matrix (IM) test of White [18] is used. Finally, this study identifies the most appropriate model that depicts the recurrent multiple failure structure of automobiles. In many research papers dealing with multiple failure time data have used plots of Cox-Snell residuals, Schoenfeld residuals and Martingale residuals. However these plots cannot be used with multiple failure times as events are dependent. Sunethra and Sooriyarachchi [13] has used White's IM [18] test as a goodness-of-fit test for the analysis of recurrent hardware failures in personal computers. In our study the goodness-of-fit of the model is tested using White's IM [18] test.

A detailed description of methods and model checking procedure are presented in Section 2 and the example is illustrated in Section 3. Section 4 gives the results of the study. Section 5 gives the conclusion and a brief discussion on the outcome of the study.

## 2.0 MATERIALS AND METHODS

Sunethra and Sooriyarachchi [13] have modeled hardware failures of personal computers (PCs) using VCPH models which has considered single type of recurrent failures. They have considered brand of PCs as the only covariate. They have used the Cox proportional hazards models discussed below to model single type recurrent failures, which are hybrid versions of the single-event Cox model and are specified in one of two ways:

$$h_{ik}(t) = h_o(t)\exp(\beta' Z_{ik}) \quad (1)$$

$$h_{ik}(t) = h_{ok}(t)\exp(\beta_k' Z_{ik}) \quad (2)$$

In these specifications, $Z_{ik}$ is a p- dimensional vector of measured covariates ( j = 1, 2,. . . p) for the $k^{\text{th}}$ event, and $\beta$ and $\beta_k$ are vectors of regression parameters to be estimated. In Equation (1), $h_o(t)$ is a non-negative baseline hazard function that is an arbitrary function of time and common to all events (i.e. $h_{ok}(t)= h_o(t)$ for $k$ =1, 2,. . . , K). In the second specification, the baseline hazard function $h_{ok}(t)$ is allowed to vary over each of the events as an arbitrary function of time. $h_{ik}(t)$ refers to the hazard function of the i$^{\text{th}}$ subject on the $k^{\text{th}}$ failure event at time t. Model (2) is known as a stratified Cox PH model and in the models discussed follow, the stratification is over $k$ failure events. The stratification is important because it allows the baseline hazard function to vary over each of the $k$ events. In this study which is an extension of above study, recurrent multiple failure modes have been considered. This results the data to be correlated among each other.

When the above usual Cox model is used to model multiple failures, which results in non-independent failure times, then the model is misspecified. Therefore it is needed to use alternative procedure to accommodate for correlation. Since then, as used by Sunethra and Sooriyarachchi[13] modified sandwich variance estimator (MSVE) is used for modeling multiple failures. Wei *et al.*[17] found that the MSVE to be a consistent estimator of the variance of the parameter estimates even under the misspecification of the dependence structure (see also Lee *et al.*[8]).

The two key components that systematically differentiate AG, WLW and PWP models are the way the risk intervals are defined with reference to the starting point and the compilation of risk sets at each distinct failure time. Risk intervals refer to the time scales used to define when a unit is at risk of experiencing a specific event. There are three possible ways of defining a risk interval and each of these describes a different substantive type of risk process. These three intervals are namely, gap time, total time and counting process risk intervals. Selecting suitable risk interval and checking the PH assumption is done using Schoenfeld's Global Test. Schoenfeld's test is used because "it has the power to detect the insufficiency of covariates in describing the relative risks and the assumption of PH" as noted by Abeysekera and Sooriyarachchi [1].

### 2.1 Prentice, Williams and Peterson (PWP) Model (Ezzell *et al.* [6])

PWP model will be used to analyze the data because of following features of the data set: gap time risk interval, event specific baseline hazard and restricted risk set. The gap time model the corresponding indicator is $Y_{ik}(t)=((x_{ik}-x_{i,k-l})\geq t)$ where $x_{ik}$ denotes the total time failure of $i^{th}$ unit at $k^{th}$ event, $x_{i,k-l}$ denotes the total time failure of $i^{th}$ unit at $k$-$1^{th}$ event and $Y_{ik}$ denotes the gap time failure of $i^{th}$ unit at $k^{th}$ event. These models attempt to account for the dependence among event times by, stratifying the model by the event number (Wei and Glidden [16]). Because the PWP models allow individuals to join new strata upon event occurrence, "the individual's baseline hazard function is allowed to change discontinuously from one non-parametrically modeled function to another" (Clayton [2]). Clearly, however, the strengths of the PWP models are that the models both take into account the explicit ordering of the failure times of a recurrent event process and allow for the estimation of event-specific parameters. However, simulations have shown the PWP models are sensitive to unobserved heterogeneity and misspecification (Lin [10], Therneau *et al.* [14], Therneau *et al.* [15]).

Depending on how the starting point of the risk interval is set, there are two variations of PWP models: PWP total time model and PWP gap time model. Here the 'total time' means the time from the start of treatment, and 'gap time' is the time from the prior event. The PWP total time model is similar to the counting process model but stratified by event. Let $h_{ok}$ be the event-specific baseline hazard for the k$^{th}$ event. PWP total time model has the form of $h_{ik}(t) = h_{ok}(t)\exp(\beta_k' Z_{ik})$ and PWP gap time model has the form of $h_{ik}(t) = h_{ok}(t - t_{k-1})\, exp(\beta_k' Z_{ik})$.

The PWP gap time model is specified with an event-specific baseline hazard function in which it allows the baseline hazard function to vary over each of the separate failure events and a restricted risk set in which it allows the given individual to contribute information to the partial likelihood function of the k$^{th}$ event at time t as long as they have not experienced the k$^{th}$ event prior to time t and are still under observation at time t. Stated in another way, subjects are considered at the risk of k$^{th}$ event prior to experiencing the (k−1)$^{th}$ event. This is the risk set of choice for the analysis of unordered data where the risks are developing simultaneously. Therefore, PWP models can be specified as follows:

$$h_{ik}(t) = h_{ok}(t - t_{k-1})\exp(\beta_k' Z_{ik}) \tag{3}$$

Lim and Zhang, [9] have defined PWP model with common parameter estimates with different baseline hazards for PWP Model. This model is defined with a gap time risk interval, a restricted risk set and with different baseline hazard function with common parameter estimates for all *k* events. This model allows the baseline hazard function to vary over each of the separate failure events but with common parameter estimates.

$$h_{ik}(t) = h_{ok}(t - t_{k-1})\exp(\beta' Z_{ik}) \tag{4}$$

### 2.2 Goodness of Fit of the Model

This study applies the IM test of White [18] for the checking of the PH model specification with multivariate failure time data because usual Kaplan Meier methods cannot be used because of correlated failure data. White's paper on inference from miss specified models presented the IM test as a test for correct model specification. The IM test is based on the sum of the mean of the cross-product of the first derivatives of the log-likelihood and the mean of the second derivatives. Both these terms are calculated at the estimated parameter values. If the model is correct, the sum of these two alternative measures should be asymptotically zero. With multivariate failure time data, three different tests can be formed. The different tests arise from the treatment given to the dependence between the univariate margins and the over-dispersion within them. According to Crouhcley and Pickels [4] a test that examines homogeneity in the effect of the covariate β, in the univariate margins and can also be used for checking model specification, takes the form,

$$\overline{D_c}(\hat{\beta}) = n^{-1}\left[\sum_i \sum_k\left[-\widehat{h_{ik}}(t)\right] + \sum_i \sum_k\left[\delta_{ik} - \widehat{h_{ik}}(t)\right]\left[\delta_{ik} - \widehat{h_{ik}}(t)\right]\right] \tag{5}$$

where $\overline{D_c}(\hat{\beta})$, follows a chi-square distribution with one degree of freedom (for one parameter estimate) under the correct model specification. Here, the subscript $k$ distinguishes each of the failure times from sample unit i. $\delta_{ik}$ is the censoring indicator taking the value 1 if failure occurs and 0 otherwise.

### 2.3 Example

In this study the time (mileage) to failure, of an automobile product is modeled considering a selection of failure types that can occur for three different types of automobiles. The three types of automobiles are denoted by *type A, type K and type R* as original brand names cannot be divulged owing to reasons of confidentiality. Six types of failures are considered and described below: Wheels/ Tyres and Vehicle Alignment, Brakes – Hydraulics/ Regulator/ Servo, Engine - Craft shaft Group/ Pistons, Ventilation / Exhaust System, Front Suspension/ Drive Shafts and Doors/ Locking Systems and each has been denoted by *failure type 1 to 6* and any other type of failure as *failure type 7*, for easy reference. Thereby, the effect of the type of vehicle and failure type occurred, on failure occurrences are evaluated over multiple failure times. Vehicles which were sold from March 2009 to April 2011 are taken in to study and failure times are recorded for those vehicles from March 2009 to May 2011. In total 2532 vehicles were contained in the sample which was used for the study. Data gathered, presented a dynamic total number of failures as vehicles which come to get repaired are only recorded and vehicles not visited are not recorded. 14 stratums (repetitive failures) will be considered, considering maximum number of failure occurrences. Each failure type will be measured for fourteen events and seven failure types will be considered for each vehicle. Therefore each vehicle will account for 98 records.

### 2.4 Descriptive Statistics

Table 1 gives the percentages of failed automobiles at each failure event for 3 different types of automobiles and it has ignored the type of failure which has occurred. When failure event increases percentage of failed vehicles have deteriorated. When it comes to seventh failure event only 10% of automobiles have failed and in the first failure event almost 95% of vehicles were failed. In all the failure events three types of automobiles have recorded almost similar percentages of failed vehicles (table 1). After 8$^{th}$ failure event failed percentage falls below 10% for all types of automobiles. Therefore it can be considered that first seven failure events play a prominent role.

**Table 1** Percentage of failed automobiles at each failure event

| Type of Automobile | Failure Event | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th | 9th | 10th | 11th | 12th | 13th | 14th |
| A | 96.90 | 87.20 | 59.50 | 40.70 | 26.70 | 17.90 | 11.40 | 6.30 | 3.50 | 2.40 | 1.80 | 1.20 | 0.80 | 0.60 |
| K | 94.90 | 79.40 | 58.50 | 39.90 | 26.40 | 16.20 | 9.80 | 5.40 | 3.00 | 1.50 | 0.50 | 0.20 | 0.20 | 0.10 |
| R | 94.30 | 74.00 | 56.90 | 41.10 | 24.80 | 16.90 | 9.30 | 5.10 | 2.70 | 0.70 | 0.10 | 0.00 | 0.00 | 0.00 |

Since mileage is not considered above, median survival mileages are considered next. Table 2 gives the median survival mileages of each automobile at each failure event and it has ignored the type of failure which has occurred. In first three failure events Model R has been outperforming A and K since median survival is higher, but after failure event three, all the models are performing evenly with slight changes.

Median survival time changes drastically from failure events 1 to 2, 2 to 3 and 3 to 4 but after fourth failure event changes occur are miniature (Table 2). It can be seen that the failure pattern of the automobile model is not similar in all the failure events. From this arises the need for investigating the effect of the automobile model and whether this effect is the same in multiple failure occurrences of different failure modes.

**Table 2** Median survival mileage (km) of each automobile model at each failure event

| Type of Automobile | Failure event | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th | 9th | 10th | 11th |
| A | 6352 | 14980 | 30740 | 51230 | 57840 | 57840 | 60870 | 53900 | 76070 | 55980 | 54950 |
| K | 6944 | 15300 | 28900 | 54250 | 53640 | 51830 | 54590 | 54590 | 54160 | 50610 | 50130 |
| R | 10350 | 19830 | 34250 | 52420 | 56820 | 60080 | 58980 | 57600 | 65610 | 64690 | |

## 2.5 Univariate Tests

Among the VCPH models described, the main differences were the definition of risk sets, definition of risk intervals and the form of the baseline hazard function. As stated by Hannan *et al.*[7], 'the programming to rearrange the data for the correct analysis requires a clear understanding of the implicit definition of the separate risk sets'. A procedure for identifying the proper structure of the risk interval suitable for the data set considered in this study is suggested by using Schoenfeld's Global Test. The type of risk interval, which does not show a significant departure from the proportionality of the hazards, will be considered as the suitable data structure for the modeling session. When the suitable risk interval is identified in this way, the allocation of varied risk sets and baseline hazard functions would be done within the modeling session. Table 3 gives the *p* values of Schoenfeld's test for PH assumption under total time and gap time.

The *p*-Values of the test for PH assumption for total time appeared to be significant at the 5% level in second and nearly significant at thirteenth and fourteenth failures. Thus, it cannot be concluded that the assumption of proportionality of hazards is prevalent in each of the failure instances under the total time risk interval. From Table 3, it can be concluded that the assumption of proportionality of hazards is prevalent in each of the failure instances under the gap time risk interval comparative with the total time risk interval.

With the findings of the univariate tests, it was decided to use the gap time risk interval as the preferred structure of data for the data set of this study. But, as these univariate tests take into account each failure event separately and ignore the correlation effects under multivariate failure times, further validation will be done in the modeling session.

**Table 3** Summary of Schoenfeld's Test for PH assumption

| Failure Event | Total Time | | Gap Time | |
|---|---|---|---|---|
| | P value | Comments | P value | Comments |
| First | 0.514639189 | Good | 0.514639189 | Good |
| Second | 0.00456312 | Departure | 0.07996794 | Slight Departure |
| Third | 0.768825138 | Good | 0.507982521 | Good |
| Fourth | 0.158666962 | Slight Departure | 0.151573513 | Good |
| Fifth | 0.366382759 | Good | 0.608255558 | Good |
| Sixth | 0.846788703 | Good | 0.346346135 | Good |
| Seventh | 0.490659474 | Good | 0.594440578 | Good |
| Eighth | 0.877616736 | Good | 0.309438867 | Good |
| Ninth | 0.435978659 | Good | 0.863757613 | Good |
| Tenth | 0.128232572 | Slight Departure | 0.131322065 | Slight Departure |
| Eleventh | 0.669887257 | Good | 0.107121513 | Slight Departure |
| Twelfth | 0.155149823 | Slight Departure | 0.062074016 | Slight Departure |
| Thirteenth | 0.074769394 | Slight Departure | 0.064661625 | Slight Departure |
| Fourteenth | 0.074511622 | Slight Departure | 0.085330520 | Slight Departure |

## 3.0 RESULTS AND DISCUSSION

In Section 2.5, it was identified that the gap time risk interval is preferred over the total time risk interval as the PH assumption was approximately valid for the former but not for the latter. Therefore, only the models that are defined with a gap time risk interval are applied in this study. The PWP model is specified with a gap time risk interval. Gap time model can be fitted with common parameter estimates and different baseline hazards or with uncommon parameter estimates with different baseline hazards.

Initially, the PWP model was fitted with uncommon parameter estimates. It was found that $10^{th}$ to $14^{th}$ failure events seem to be insignificant and was considered as a common event. A model was developed with 80 parameter estimates. Since then, it is more worthwhile to check whether all events can be estimated with a common parameter without losing the accuracy. Therefore, PWP model is fitted with common parameters for and appropriateness of it is checked using White's IM test. The chosen model can be specified as in equation 6 along with parameter estimates (Table 4).

$$h_{ik}(t) = h_{0k}(t - t_{k-1}).\exp(\beta_1 x_1 + \beta_2 x_2 + \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 + \alpha_4 y_4 + \alpha_5 y_5 + \alpha_6 y_6) \quad (6)$$

where $x_1$- type A automobile relative to type R automobile, $x_2$- type K automobile relative to type R automobile $y_1, y_2, y_3, y_4, y_5$ *and* $y_6$ are failure type 1, 2, 3, 4, 5, 6 relative to failure type 7 respectively.

The *p*- value of the Wald test in the above model indicates that the effect of the model of automobile and failure types is highly significant (*p*- value <0.0001).Both type *A* and *K* automobile's instantaneous failure rates are 1.574 times and 1.395 times higher than the type *R* automobile, respectively.

**Table 4** Parameter estimates of the fitted final model

| Included Covariates | Parameter Estimates | | Hazard Ratio | Std Error | 95% Confidence Limit | | *P*-Value |
|---|---|---|---|---|---|---|---|
| Type of automobile | A | 0.4534 | 1.574 | 0.031 | 1.481 | 1.672 | <.0001 |
| | K | 0.33313 | 1.395 | 0.02573 | 1.327 | 1.467 | <.0001 |
| Failure type occurred | 1 | 2.97105 | 19.512 | 0.09399 | 16.23 | 23.459 | <.0001 |
| | 2 | 2.84304 | 17.168 | 0.0943 | 14.271 | 20.653 | <.0001 |
| | 3 | 2.92342 | 18.605 | 0.0941 | 15.471 | 22.373 | <.0001 |
| | 4 | 2.40321 | 11.059 | 0.09572 | 9.167 | 13.341 | <.0001 |
| | 5 | 1.62939 | 5.101 | 0.10025 | 4.191 | 6.208 | <.0001 |
| | 6 | 0.61449 | 1.849 | 0.11379 | 1.479 | 2.311 | <.0001 |

Instantaneous failure rates of different failure types when compared with failure type 7(any other type of failure that isn't captured under six failure types) of three automobile types are identified in Table 5. For all three vehicle types wheel failures are more frequently occurred.

However, further assessment of the fit of the model is required before concluding the final model to model the multiple failure occurrences of automobiles. The IM test is carried out on above model to find out the goodness of the model specification. The $\overline{D_c}(\hat{\beta})$ statistic was computed for the above model, which assumes common effects for all the events considering all the parameter estimates. $\overline{D_c}(\hat{\beta})$ Statistic was 5.8167 on 8 degrees of freedom (since model estimates eight parameter estimates). A *p*-value of 0.66775 indicates that the model is correctly specified. Therefore it is reasonable to assume that the above model suits significantly well to model the instantaneous failure rate of automobiles in all the 14 events.

**Table 5** Instantaneous failure rate of different failure types when compared with failure type 7

| Type of Automobile | Failure Type | | | | | |
|---|---|---|---|---|---|---|
| | Wheel | Brake | Engine | Ventilation, Exhaust System | Front Suspension, Drive Shafts | Doors, Locking System |
| R | 19.5 | 17.16 | 18.6 | 11.05 | 5.1 | 1.84 |
| A | 30.7 | 27.01 | 29.27 | 17.4 | 8.02 | 2.9 |
| K | 78.75 | 69.29 | 75.09 | 44.63 | 20.58 | 7.46 |

## 4.0 CONCLUSION

The analysis carried out in this study showed that the automobile model and failure types occurred affects failure event timings. Further it was identified that this effect of the automobile model remains constant over the multiple failure occurrences. The VCPH models discussed in this paper give an appropriate analysis of multiple type recurrent failure occurrences in which dependence between the failure events is captured. The conventional method of checking the goodness-of-fit of a Cox proportional model (Cox–Snell residuals, martingale residuals, etc) is not appropriate in the event of multiple failure occurrences due to the dependence among failure events. In contrast to the usual methods that have been frequently used to check the goodness-of-fit of PH models, the IM test proposed by White[18], which is preferred in the presence of multiple failures is used in this study.

## Acknowledgement

The authors are grateful to Dr. Dilhari Attygalle, Head, and staff of the Department of Statistics, University of Colombo for their courteous help and friendliness. In addition, special thanks to Mr. G. S. Amarasinghe whose familiarity with the automobile area was helpful to us in accessing data.

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