Proceedings of the Technical Sessions, 23 (2007) 7-12
Institute of Physics – Sri Lanka

3D Modelling of Electrical Discharges

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ABSTRACT

Long electrical discharge patterns in 3D were simulated using a stochastic dielectric breakdown model. The 2D stochastic model introduced by the Niemeyer et al. was modified to simulate the electrical breakdowns in 3D. To compare and classify the simulated breakdown patterns, fractal techniques were used. The simulated electrical breakdown patterns dependent highly on 'η', the exponent of the cell potentials defined in the breakdown probability equation. For small η values ($\eta \approx 2$), highly complex discharge patterns with many branches having fractal dimension of 1.66 ± 0.02 was observed. When 'η' increases ($\eta \approx 10$), the growth patterns effectively loose their fractal structure and become a curve with dimension 1. In each 3D simulated pattern, 2D vertical projections on XZ plane and YZ plane were analyzed. By studying the fractal dimension of the projections, most suitable η value for simulating discharge patterns can be deduced.

1. INTRODUCTION

To simulate electrical discharge patterns such as lightning in the atmosphere or development of surface discharges on insulating materials, two models are widely used, namely, the Dielectric Breakdown (DB) model and the Diffusion Limited Aggregation (DLA) model. DB model provides theoretical background well suited for the formation of long electrical discharges than the DLA model. Many studies are available in literature on the simulation of 2D discharge patterns using DB model. In this work, DB model has been extended to simulate long electrical discharges in 3D.

The initial stochastic DB model was developed to describe the geometric structure of 2D gas discharges by Niemeyer et al [1]. The model was developed as a growth pattern, which finds the new growth sites according to the magnitude of the potential in the nearest neighbours. The basic assumption is the growth probability of conducting structure depends on the local electric field. They have shown that the average fractal dimension of simulated discharge patterns were 1.75 ± 0.02.

Satpathy [2] simulated dielectric breakdown in 3D by using an improved DB model. In his study, two cases were analyzed, such as 2D patterns in a 3D Laplace field and 3D patterns in a 3D Laplace field. Simulated patterns were represented as 3D discharge patterns. Average fractal dimension of the highly branched discharge patterns have been reported as 2.48 ± 0.06.

Barclay et al [3] used improved DB model to simulate electric discharge patterns. They extended their study from electrical discharges in 2D to electrical discharges in 3D in order to check whether the patterns of behaviour established for planar geometry remain valid for simulation in a more realistic geometry. They reported average 3D fractal dimension value of highly branched patterns as 2.5 and typical patterns as 1.7.

Sanudo et al [4] developed three dimensional DB model based on the stochastic fractal model proposed by Niemeyer et al. They reported average fractal dimension in 3D as 1.51 and 2D fractal dimension for the vertical projections (XZ & YZ) as 1.34. These results agreed with previously published results only at higher value of the exponent.

Kudo [5] analyzed electrical treeing in experimental and simulated patterns using fractal dimension techniques. For computer simulation, Kudo used Niemeyer's stochastic DB model with moderate improvements such as adding a critical potential to initiate a breakdown. He analyzed 3D simulated discharge patterns and their 2D projections and the relationship between the projections.

2. METHODOLOGY

2.1 Three Dimensional Dielectric Breakdown Model

In this work, two dimension stochastic DBM has been extended to simulate 3 dimensional electrical discharge patterns within a volume of 50×50×50 cubic lattice. The initial boundary condition for the 3D growth structure is shown in the figure 1. The grey dot represents the cell having negative potential ($\phi=0$) at the top of cubic lattice structure which is the starting point of the pattern. Black dots represents the positive potential ($\phi=1$) at the bottom plane of the cubic lattice. The gap distance between the negative potential and positive potential is equal to 50 lattice units.

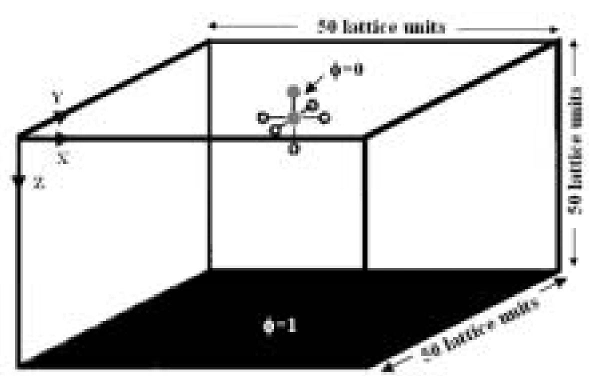

Figure 1: The initial boundary condition in 50×50×50 cubic lattice

The pattern develops stepwise from negative potential to positive potential depending on the cell potentials of nearest neighbours. Cell potential of each lattice point can be calculated by solving the Laplace equation numerically, over the 3D grid (equation 1) subjected to the initial boundary conditions.

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad \text{-- (1)}$$

The finite difference method can be used to calculate the numerical solution of the 3D Laplace equation [6]. By using nearest neighbours in *x*-direction, *y*-direction and *z*-direction, potential at an arbitrary cell (*x*,*y*,*z*) can be calculated using the formula given in equation 2.

$$\phi(x,y,z)=\frac{1}{6}\left[\phi(i+1,j,k)+\phi(i-1,j,k)+\phi(i,j+1,k)+\phi(i,j-1,k)+\phi(i,j,k+1)+\phi(i,j,k-1)\right]$$
-- (2)

In DB model, the growing pattern, step by step, searches for the next possible extension out of all possible lattice cells, depending on the magnitude of cell potentials [1]. In Figure 1, open circles show the selected cell configuration for the first step. In this work, the diagonal cells were not included as nearest neighbours. This was done in order to compare our work with the previously published results. Reader should be aware that by inclusion of diagonal cells, fractal estimates generally tends to produce a lower fractal dimension values. The selection of the growth site was decided according to the probability distribution given in equation 3.

$$Pi=\frac{\phi_i^\eta}{\sum_{i=1}^{n}\phi_i^\eta} \qquad \text{-- (3)}$$

The selected point is added to the growing pattern and assign the same potential as the potential of the growing pattern (ϕ= 0). After step 1, cell potential of each lattice cell is recalculated to introduce the effect due to newly added cell to the growth pattern. For the 2nd step there are a total of 9 possible cells to select from, i.e. 5 new cells due to the newly added cell and 4 cells remaining from the previous configuration. The DB model searches next possible cell to expand out of all 9 cells depending on their potential. Thus, at each step, one point is added to the pattern according to the probability *P*.

The exponent 'η' is a model parameter that modulates the randomness of the process and it describes the relationship between the local field and the breakdown probability. Thus, 'η' controls the appearance of the branches (complexity) in patterns. When 'η' increases number of branches in the pattern decreases. The most time consuming part of the simulation was solving the Laplace equation iteratively. To increase the computational speed, initial iterations were applied to a 20x20x20 cell configuration centred on the newly added cell which was expanded to the whole lattice until convergence.

2.2 Estimating fractal dimension

Sandbox method was used to calculate the fractal dimension of the generated discharge patterns as well as the projections of the patterns. To estimate the fractal dimension of the generated 3D patterns, a square box of size '*L*' is formed on the pattern and the mass of the pattern found within the box is evaluated. To estimate the fractal dimension of the 2D projections, squares of size '*L*' is formed on the projected pattern and the mass of the pattern found within the square was evaluated. The mass can be evaluated by counting the number of lattice points within the box or the square. The average mass *M(L)* is obtained for different sizes '*L*'. Sandbox fractal dimension *D* is the exponent that expresses the scaling of the mass with its size as shown in equation 4.

$$M(L) \propto L^{D2} \qquad \text{--- (4)}$$

Generally, fractal dimension can be found by calculating the gradient of a double log plot.

3. RESULTS

The simulated 3D electrical discharge patterns and their 2D projection on XZ and YZ planes are shown in Figure 2 for η=2 and η=10. The effect of the model parameter η can be seen in these simulated patterns. It controls the diffusion of the branches in discharge patterns. Estimated Sandbox fractal dimension clearly shows this behaviour. The estimated fractal dimension values are categorized under Table 1 for η=2, 4, 6, 8 and 10. This result agrees with the past studies on η for 3D and 2D patterns.

Table 1: 3D and 2D fractal dimension values for different η values

Fractal Dimension	η=2	η=4	η=6	η=8	η=10
3D	1.66±0.02	1.59±0.11	1.39±0.06	1.14±0.1	1.05±0.08
2D – XZ plane	1.42±0.04	1.33±0.05	1.26±0.04	1.06±0.06	1.00±0.05
2D – YZ plane	1.44±0.02	1.39±0.07	1.21±0.08	1.05±0.06	1.01±0.06

It can be seen that, fractal dimension of 2D projections on XZ and YZ planes are nearly equal. One would naively expect this result. For example, we would expect fractal nature of the natural lightning flashes to be the same in any direction. However, reduction in the fractal dimension between the 3D and 2D projections were seen in data. The same result has been confirmed by another study [3]. In a recent study [7], 2D lightning discharge patterns have been simulated using DBM for three η values, η=1, 2 and 3. When 2D simulated patterns and 2D projections were compared, difference between each other for the same η value was seen. A higher η value is required in 2D projections of the 3D simulated patterns to obtain the same fractal dimension values as 2D simulated patterns.

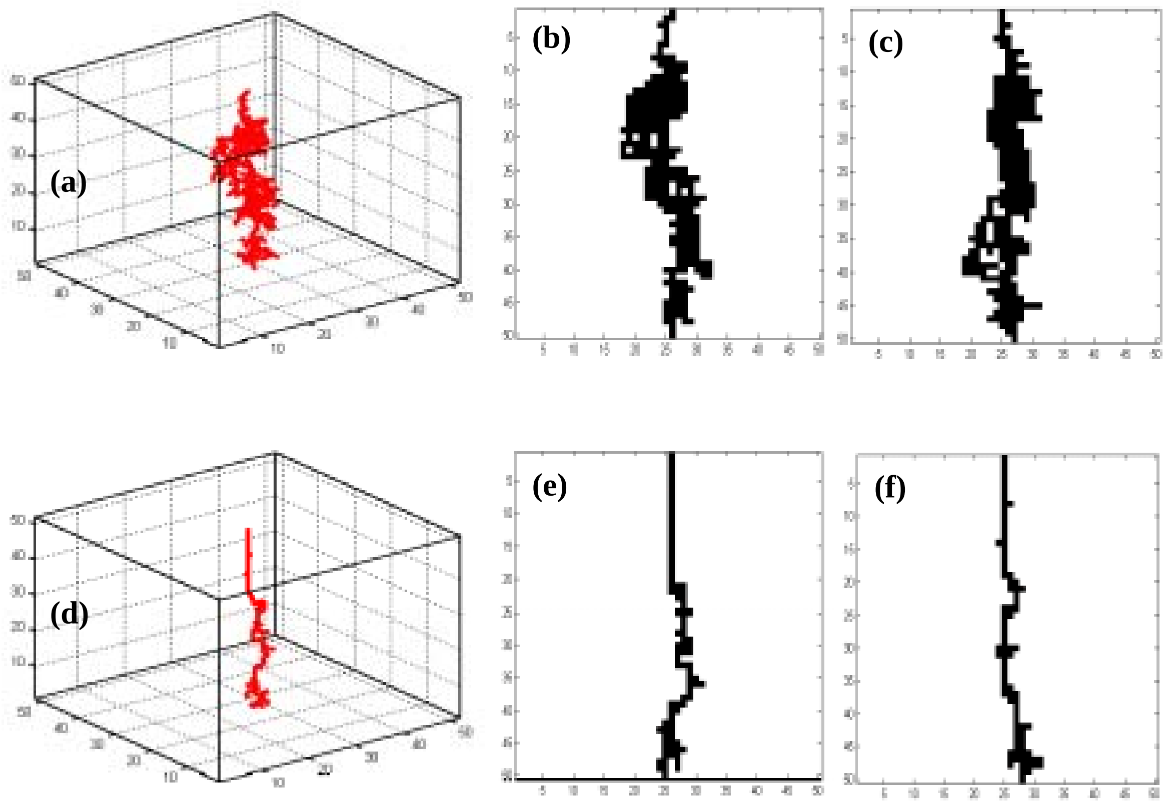

Figure 2: (a) 3D simulated pattern when η=2, (b) 2D projection in XZ-plane (c) 2D projection in YZ-plane, and, (d) 3D simulated pattern when η=10, (e) 2D projection in XZ-plane (f) 2D projection in YZ-plane

When 3D fractal dimension and 2D fractal dimension of their projections were compared, a strong linear relationship with a squared correlation coefficient of 0.998 was observed. This relationship is show in the equation 5.

$$D_{2D\ projection} = 0.69 \times D_{3D} + 0.27 \qquad (5)$$

Previous work reported that the average 2D fractal dimension of long laboratory sparks to be 1.39 [7]. According to the simulated results it is seen that the DB model can reproduce 3D laboratory discharge patterns when $\eta \approx 4$. Since there is no difference in simulating long sparks and natural lightning, the DB model can be extend to compare with the 3D fractal dimension of natural lightning flashes.

4. DISCUSSION AND CONCLUSIONS

The DB model provides the basic theoretical foundation for simulating the stochastic breakdown of electrical discharges. Since the discrete lattice potentials mimic the possible electric field effects generate by the space chargers in the atmosphere, the DB model is more realistic model to simulate long electrical breakdowns than DLA model. In this work, the stochastic DB model introduced by the Niemeyer et al. was extended to study the 3D electrical discharge patterns. The fractal dimension was used to analyze the complexity of simulated discharge patterns in 3D as well as 2D vertical projection of the same.

The effect due to the model parameter 'η' was very clear in the simulated discharge patterns which can be quantified through fractal dimension estimations. It was seen that the role of 'η' controls the probability of appearance of branches in the patterns. When 'η' increases (η=10), the growth patterns effectively loose their fractal structure and become a curve with dimension 1. By comparing the fractal dimension of the experimental laboratory sparks and 2D projection of the simulated patterns, best 'η' value can be found to simulate long electrical discharges in 3D. The value that approximates the experimental observations is found to be η≈4. The same idea can be extended to simulate lightning flashes.

The major disadvantage in the 3D stochastic model is the execution time which is highly dependent on the size of the cubic lattice chosen. This is directly linked to the solving of Laplace equation iteratively for each step.

ACKNOWLEDGEMENTS

Financial assistance given by the IPPS of the International Science Programs, Uppsala University (SRI 01/3), and the National Science Foundation, Sri Lanka for the research grant RG/04/E/06 are greatly acknowledged.

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