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# Fractal dimension of long electrical discharges

Dulan Amarasinghe  $^{\rm a}$ , Upul Sonnadara  $^{\rm a,\,*}$ , Marcus Berg  $^{\rm b}$ , Vernon Cooray  $^{\rm b}$ 

- <sup>a</sup> Department of Physics, University of Colombo, Colombo 3, Sri Lanka
- <sup>b</sup> Division for Electricity and Lightning Research, Uppsala University, Sweden



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#### ABSTRACT

The fractal dimension of 500 mm long electrical discharges is presented by analyzing a set of photographic images. Three popular fractal dimension estimation techniques, box counting, sandbox and correlation function methods were used to estimate the fractal dimension of the discharge channels. To remove the apparent thickness due to varying magnitudes of current in the discharge channels, edge detection algorithms were utilized. The estimated fractal dimensions for box counting, sandbox and correlation function for long laboratory sparks were  $1.20 \pm 0.06$ ,  $1.66 \pm 0.05$  and  $1.52 \pm 0.12$  respectively. Within statistical uncertainties, the estimated fractal dimensions of positive and negative polarities agreed very well.

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### Introduction

Estimating the tortuosity of the irregular paths taken by the discharge channels is necessary when determining the extent to which the complex paths contribute to the electromagnetic radiation or even attributing to the type of electrical breakdowns. In the past, a statistical method which was based on the idea of a path taken by a random walker has been utilized to quantify the channel tortuosity of natural lightning channels, triggered lightning channels and long laboratory sparks [1-3]. The main disadvantage of this statistical technique is the need to accurately identify the direction changes of the linear sections of the channel along the path of the discharge channels. The identification could become quite difficult and lead to errors when branches are present with varying degrees of luminosity which may appear to be close to one another in a 2D view, especially for complex discharge channels. It is virtually impossible to apply this statistical method to find the channel tortuosity of surface discharge patterns or electrical trees. Thus, this technique is useful only for discharge channels having no branches or for less complex discharge channels with a few clearly visible branches.

As an alternative, fractal techniques can be applied to characterize the channel tortuosity in electrical discharge channels. Radom and complex patterns have been successfully characterized by using fractal dimension. Niemeyer et al. [4]. have reported a fractal dimension for two-dimensional branched gas discharges as

1.7, which is normally considered as the common value for the fractal dimension of electrical discharges that are highly branched. By analyzing a set of lightning photographs, Tsonis [5] reported a fractal dimension of natural lightning as ~1.34. Most of the recent work in this area has concentrated on the development of stochastic dielectric breakdown models by computer simulations in two-dimensions or three-dimensions to estimate the fractal dimensions of electrical discharges [6–9]. Thus, it is important to document fractal dimensions of long electrical discharges since collaborative experimental measurements are scarce in the literature. It would also be interesting to compare the fractal dimension of triggered lightning flashes with natural lightning flashes given the fact that the reported tortuosity values are significantly different for the two processes [1,2]. However, this is out of the scope of the work presented in this paper.

When estimating fractal dimensions, there are a number of different algorithms that can be applied to electrical discharges [10]. Some of these algorithms depend on the spatial distributions of fractal patterns and others depend on the time-dependent nature of fractal development. Due to the different definitions of the fractal dimension, different values for the same fractal structure could be observed. One may refer the work done by Barclay et al. [11] and Kudo et al. [12] in this area.

In this work, the Box counting, Sandbox and Correlation function methods were utilized to estimate the fractal dimension of 500 mm long electrical discharges. Several popular edge detection algorithms were utilized in pre-processing the photographic images in order to remove the apparent thickness of the discharge channel caused by the magnitude of the flowing currents through different branches which may affect the estimation of fractal

<sup>\*</sup> Corresponding author. Tel./fax: +94 11 2584777. E-mail address: upul@phys.cmb.ac.lk (U. Sonnadara).