ABSTRACT

Algebras of monads over the category of sets and over the category of topological spaces

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Algebras of monads over ENS the category of sets and over TOP the category of topological spaces are studied in this thesis.

There are sixteen induced monads $\mathcal{F}_t$ over ENS depending on t, obtained from sixteen adjunctions $P_t^{\mathrm{op}} \dashv G_t : \mathrm{ORD}_t^{\mathrm{op}} \longrightarrow \mathrm{ENS}$, where t is a list of some or all of four operations on ordered sets, denoted by $\wedge, \vee, \top, \bot$. Categories of $\mathcal{F}_t$-algebras $\mathrm{ENS}^{\mathcal{F}_{\wedge\vee\top\bot}}$, $\mathrm{ENS}^{\mathcal{F}_{\wedge\top\bot}}$, $\mathrm{ENS}^{\mathcal{F}_{\wedge\top}}$ and $\mathrm{ENS}^{\mathcal{F}_{\top\bot}}$ are the categories of compact Hausdorff spaces, continuous semilattices, continuous lattices and completely distributive lattices respectively. All other categories of $\mathcal{F}_t$-algebras are variants of these.

Four monads over TOP, the prime closed filter monad $\mathcal{W}_p$, the prime open filter monad $\mathcal{H}_p$, the proper closed filter monad $\mathcal{W}_0$ and the proper open filter monad $\mathcal{H}_0$, where $\mathcal{W}_p$ and $\mathcal{H}_p$ are obtained by preimage adjunctions between LAT the category of lattices and TOP considering closed and open sets respectively and $\mathcal{W}_0$ and $\mathcal{H}_0$ are obtained by preimage adjunctions between MSL_0 the category of $\mathrm{ORD}_{\wedge\top}$ and TOP considering closed and open sets respectively are also studied. Categories of $\mathcal{W}_p$-algebras and $\mathcal{H}_p$-algebras are compact ordered spaces. The categories of $\mathcal{W}_0$-algebras and $\mathcal{H}_0$-algebras are isomorphic with the category of algebras $\mathrm{ENS}^{\mathcal{F}_{\wedge\top}}$ of the proper filter monad $\mathcal{F}_{\wedge\top}$ over ENS.